



## VISUALIZATION

*Thinking strategies to enhance skills of analysis, creativity, and problem solving.*

## VISUALIZING THE HANDSHAKE PROBLEM

**Brief Description of the Lesson:** The students will investigate various forms of the handshake problem by using visualization problem solving strategies like drawing, acting it out and making a chart/table.

**SOL/POS Objective:** *(List number and specific objective.)*

### Math 5.1.3

Students will use problem solving approaches to understand concepts and skills. Students pose problems; solve routine, non-routine and multi-step problems.

#### 5.1.3.1

Develop and apply operations and strategies (e.g., act it out, build a model, draw a picture or diagram, guess and check, make a chart or table, make a list, make a graph, use a pattern, use logical reasoning, solve a simpler problem, work backward) to solve a wide variety of routine and non-routine problems.

#### 5.1.3.2

Solve problems using logical procedure (a plan).

#### 5.1.3.6

Identify information that is available but not needed. Identify additional information needed to solve a problem.

#### 5.1.3.8

Share and explain (verbalize/ record) thinking during and after solving a problem. Verify and interpret results with respect to the original problem situation to determine if answers are reasonable.

### Instructional Strategy:

Nonlinguistic Representations: Students explore a mathematical problem through visualization.

**Materials:** Handshake Problem handout

### Enduring Understanding:

- The process of visualization aids in problem solving situations.
- Spatial visualization is a critical skill for mathematical understanding.
- Visualization provides visual spatial cues which strengthen connections and aids in the retention of knowledge.
- Visualization is an effective math strategy that builds spatial intelligence.
- Problem solving strategies like drawing and acting it out can be helpful for visualizing complex problems.

**Introduction/Essential Questions:** *(What influences your reaction to an issue or a problem?)*

- Why should we use visualization in mathematics?
- When does visualization help you in mathematical situations?
- What other ways can visualization be used in mathematics?
- What influences your mathematical visualization?



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- How does visualization foster creativity and problem solving?

**Teach and Explore Strategy:** *(Steps in teaching the process and exploring applications)*  
Prior to teaching this lesson, be sure to have taught or reviewed the introductory thinking process lesson. Help focus students' thinking by reminding them of the metacognitive component of this lesson.

- **Rationale:** Visualization is the conscious act of forming mental images of pictures of something that is not actually present to the senses. Mathematical concepts are based on mental images. The ability to visualize is an essential element of mathematical understanding. Drawing, Making a Chart and Acting It Out are useful strategies to promote the visualization of problems and concepts.
- **Introduce mathematical visualization:** Describe this scenario orally to the students. Tell them to close their eyes to visualize the events.
  - *Two business personnel meet at a local restaurant for lunch. They greet each other with a handshake. How many handshakes were exchanged? (1) 5 minutes later another person arrives at the table and greets the others with a handshake. How many more handshakes were exchanged? (2) Therefore, 3 people exchange 3 handshakes.*
- Bring the visualization to life by having 3 student act out the scenario. Expand the scenario to include two business people meeting two clients for lunch (6 handshakes)
- Ask if anyone can think of another good strategy for solving visualization problems. Ask for a volunteer to demonstrate the above scenarios using drawing as a strategy.
- **Exploration and Discovery:** Distribute the handshake problem handout. Allow students to work independently or with a partner. Allow for exploration and discovery by making the students struggle with the problem on their own.
- **Guided Practice:** Lead a discussion half way through the work time. Ask students who are successfully visualizing the problem to share their strategy without giving the answer. Encourage the students to try new strategies of visualization. Students who finish early should create their own scenario and solve it.
- **Closure:** Allow students to share their approach to solving the problem. Encourage students to share the various paths to the solution. Ask the students to justify why they believe their answer is correct. As a class, come to a consensus about the correct answer. Students may agree or disagree with answers, even their own.
- Discuss how visualization is important for mathematicians. Brainstorm ways to develop spatial intelligence.

**Assessment Evidence:** (Discussion, teacher observation, completed product, student reflection...)

- Teacher observations
- Completed products and sharing

**Metacognition:**

- How can the process of visualization help you solve problems?



VISUALIZATION

- How can you develop your spatial visualization intelligence?
- How does visualization help you discover patterns?
- What thinking is involved in using the strategy of visualization?

**Extensions:**

- Create new handshake problems to solve.

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## Teacher Background Knowledge

These types of problems occur around a basic pattern. Students can act it out, draw pictures and create charts to visualize the information.

# of People	# of Handshakes
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28

**The algebraic equation of this type of problem is  $n^2-n/2$**

(The number of hand shakes squared, minus the number of people, divided by 2.)

**OR another version of the algebraic equation is  $n(n-1)/2$**

(Multiply whatever the number is by one less, then divide by 2.)

### Handshake Problem Student Handout-KEY

If you had eight people in a group and each one had to shake everyone else's hand one, how many handshakes would take place?

**28 handshakes**

Suppose you shook hands with everyone in this math class. How many handshakes would occur?

**Answer varies according to the number of students in the class. Use the pattern or formula to check student answers.**

You are in a roomful of 35 people. Everyone is asked to shake hands with everyone. How many handshakes will there be? How can you figure this out? What strategies will you use?

**595 handshakes**



## The Handshake Problem

Student Name: \_\_\_\_\_

*Choose a strategy to visualize and solve the problems.  
Try different strategies! Which strategy helps you visualize the best?*

If you had eight people in a group and each one had to shake everyone else's hand one, how many handshakes would take place?

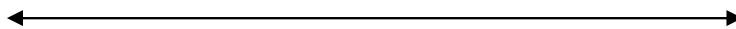
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You are in a roomful of 35 people. Everyone is asked to shake hands with everyone. How many handshakes will there be? How can you figure this problem out? What strategies will you use?

Create your own handshake problem on the back!

Challenge: Can you discover the formula to solve for any number of people?

Super Challenge: Can you think of a way to use the diagonals of geometric figure with vertices to visualize this type of problem?



Process Lesson: Visualization